Math 2D Quiz 3 Afternoon - February 4th

Please put name and ID on *both* sides for grading and redistribution!

Show all of your work. *There is a question on the back side.

1. (a) Sketch the surface $-x^2 + y^2 + z^2 + 2x + 2y - 4z + 3 = 0$.

~~It is okay if your sketch is rough, but~~ be sure to clearly note any translations.

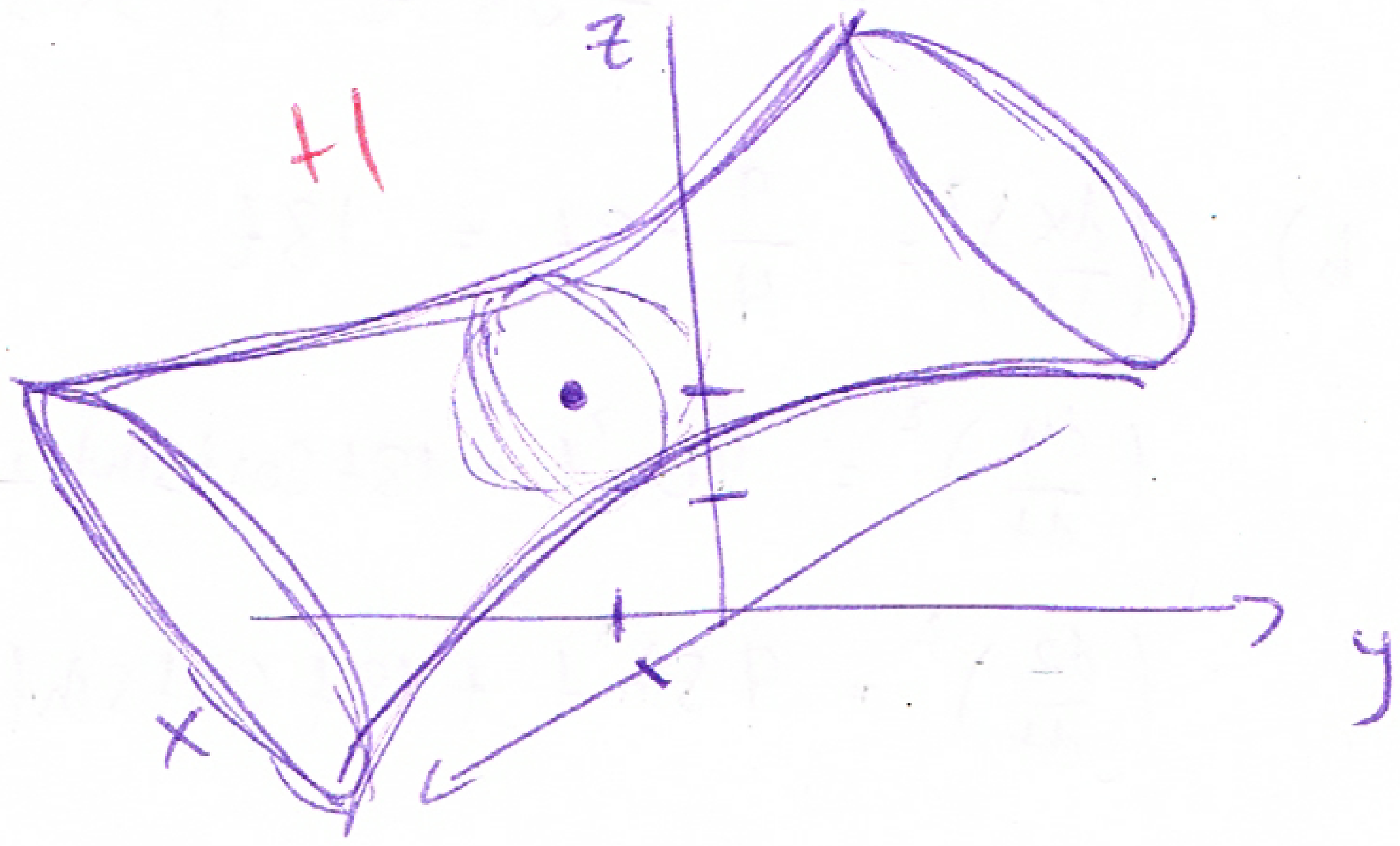
1^{st} complete the squares.

$-(x-1)^2 + (y+1)^2 + (z-2)^2 + 3 = -\cancel{1} + \cancel{1} + 4$ +1

we get $\boxed{-(x-1)^2 + (y+1)^2 + (z-2)^2 = 1}$

↳ 1-sheet Hyperboloid in x-direction "center" at $(1,-1,2)$

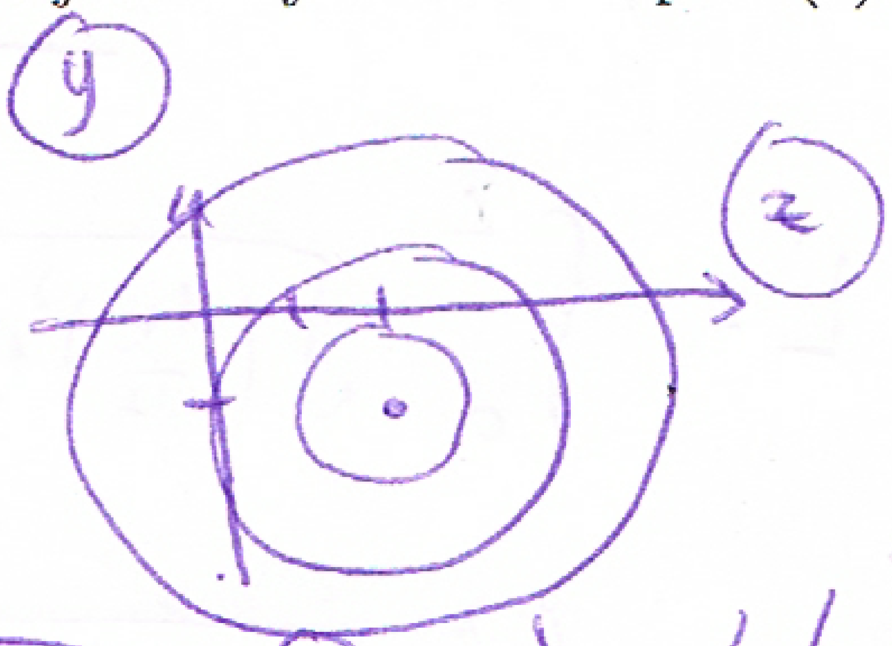
+1



(b) Also, sketch the traces in $x = k, y = k, z = k$ respectively.
(You don't have to exactly translate your traces to your sketch; it's just for you to check part (a).)

$x = k$: $(y+1)^2 + (z-2)^2 = 1 + (k-1)^2$, circles

+1



$y = k$: $-(x-1)^2 + (z-2)^2 = 1 - (k+1)^2$

+1 hyperbolas

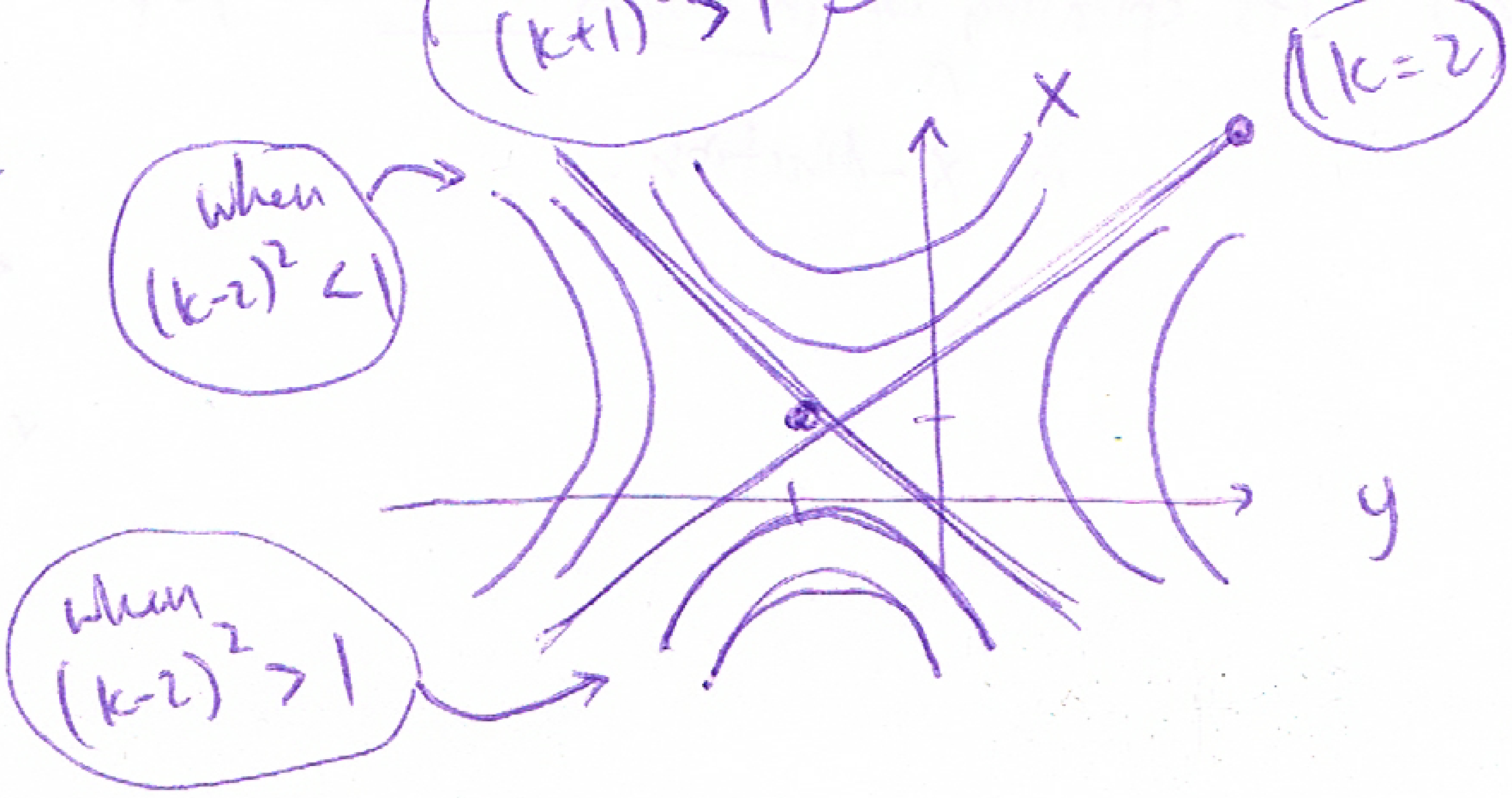
☆ There's a change when $(k+1)^2 = 1$

+1

$z = k$: $-(x-1)^2 + (y+1)^2 = 1 - (k-2)^2$

☆ There's a change when $(k-2)^2 = 1$.

(Again, hyperbolas)



2. Let $\mathbf{r}(t) = \, < x(t), y(t), z(t) > \, = \, < \sqrt{8}\, t^{3/2},\ 3t\cos(t),\ 3t\sin(t) >$ parameterize a curve.
(a) Find $\mathbf{r}'(t)$.
(b) Using part (a), compute the three quantities $(\frac{dx}{dt})^2$, $(\frac{dy}{dt})^2$, and $(\frac{dz}{dt})^2$.
(c) Compute the length of the curve starting from the point $(0,0,0)$ and ending at $(8, 6\cos(2), 6\sin(2))$.
(d) **Optional** Graph the curve for $t \geq 0$. If you lose any points from (a)(b)(c), part (d) can give you 1 extra credit point. Indicate the direction with arrows as t increases.

a) $\vec{r}\,'(t) = \, < \frac{3}{2}\sqrt{8}\, t^{1/2},\ 3\cos t - 3t\sin t,\ 3\sin t + 3t\cos t >$ +1

b) $\left(\frac{dx}{dt}\right)^2 = \frac{9}{4} \cdot 8t = 18t$

$\left(\frac{dy}{dt}\right)^2 = 9\cos^2 t - 18t\cos t\sin t + 9t^2\sin^2 t$ +2

$\left(\frac{dz}{dt}\right)^2 = 9\sin^2 t + 18t\cos t\sin t + 9t^2\cos^2 t$

c) First, $(0,0,0) \Rightarrow \underline{t_0 = 0}$ and $(8, 6\cos 2, 6\sin 2) \Rightarrow \underline{t_f = 2}$.

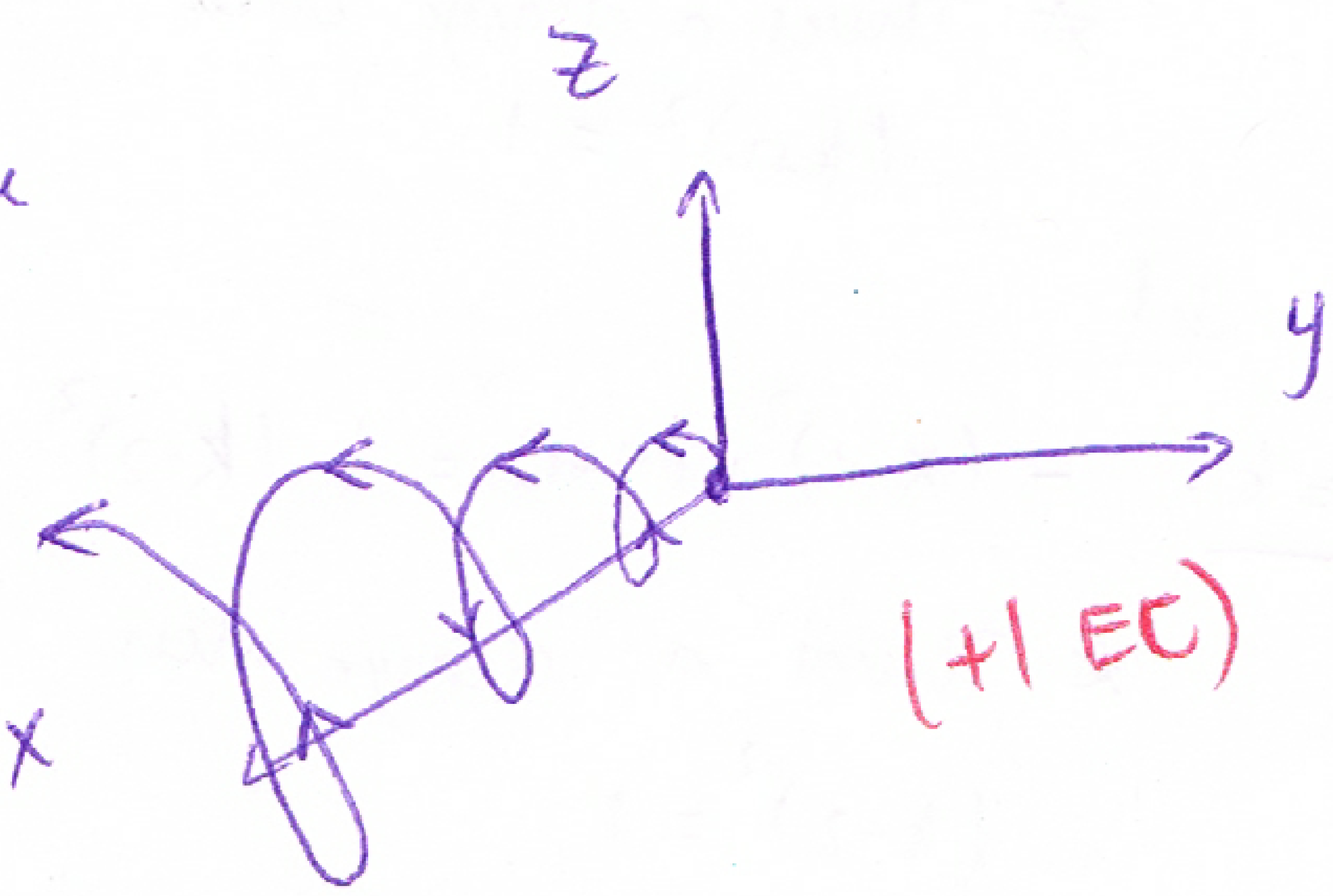
Then, $L = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$

★ utilize $\cos^2 t + \sin^2 t = 1$ and cancel the $\pm 18t\cos t\sin t$!

$= \int_0^2 \sqrt{18t + 9 + 9t^2}\, dt = \int_0^2 \sqrt{9(t+1)^2}\, dt$ +2

$= 3\int_0^2 (t+1)\, dt = 3\left[\frac{t^2}{2} + t\right]_0^2 = 3(4) = \boxed{12}$

d) It's spiraling w/ increasing radii in yz plane in x-direction.



(+1 EC)